Master Formula for the Three-Gluon Contribution to Single Spin Asymmetry in Semi-Inclusive Deep Inelastic Scattering

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Abstract

We derive a "master formula" for the contribution of the three-gluon correlation function in the nucleon to the twist-3 single-spin-dependent cross section for semi-inclusive deep-inelastic scattering, $ep^{\uparrow} \to eDX$. This is an extension of the similar formula known for the so-called soft-gluon-pole contribution induced by the quark-gluon correlation function in a variety of processes. Our master formula reduces the relevant interfering partonic subprocess with the participation of the three gluons to the Born cross sections for the $\gamma^* g \to c\bar{c}$ scattering, which reveals the new structure behind the twist-3 single spin asymmetry and simplifies the actual calculation greatly. A possible extension to higher order corrections is also discussed.

1 Introduction

Single (transverse) spin asymmetries (SSA) in high-energy inclusive processes, such as semi-inclusive deep inelastic scattering (SIDIS), Drell-Yan process, and hadron production in pp collision, appear as a consequence of multi-parton correlations in the hadrons, which did not show up in the naive parton picture for high-energy scattering. SSA reveals the multi-parton correlation most unambiguously as a leading twist-3 observable in the framework of collinear factorization which is valid to describe hadron production with large transverse momentum $P_T \gg \Lambda_{\rm QCD}$. [1] Among those correlations, the effect of quark-gluon correlations have been widely studied in the literature and our understanding on the mechanism of SSA has made a great progress [1]-[16]. There are also some studies on the contribution to SSA from the purely gluonic correlation in the nucleon [17, 18, 19, 20] and the multi-parton correlations in the fragmentation functions [21, 22].

In our recent paper [19], we have established the formalism for calculating the twist-3 single-spin dependent cross section induced by the three-gluon correlation functions in the transversely polarized nucleon. There, we clarified a complete set of the gauge-invariant three-gluon correlation functions and derived the corresponding cross section for the D-meson production in SIDIS, $ep^\uparrow \to eDX$, which is relevant to probe the purely gluonic correlation leading to SSA. [2] In particular, we have proved the factorization property of the twist-3 cross section and given a detailed prescription of expressing the cross section in terms of the gauge-invariant three-gluon correlation functions. Our result differed from the previous study [17, 18], and we clarified why the previous result needs to be corrected.

The partonic cross section in this formalism is given as a pole contribution of an internal propagator in the hard part, reflecting the naively T-odd nature of SSA. The pole forces one of the gluon lines in the three-gluon correlation functions to be soft, and hence the pole is called the soft-gluon-pole (SGP). For the SGP contribution from the quark-gluon correlation functions, it has been shown in [10, 11] that the corresponding twist-3 hard cross sections for $ep^\uparrow \to e\pi X$ and $p^\uparrow p \to \pi X$ have a simple relation with the twist-2 unpolarized hard cross section for the same processes. This connection greatly facilitates the actual calculation and makes the structure of the SGP contribution transparent.

The purpose of this paper is to show that the whole three-gluon contribution to $ep^\uparrow \to eDX$ can also be obtained from the Born cross sections for the $\gamma^* g \to c\bar{c}$ scattering. In particular, we will show that some of the twist-3 cross section is completely determined by the gluon-contribution to the twist-2 unpolarized cross section.

The remainder of this paper is organized as follows: In section 2, we briefly recall a complete set of the twist-3 three-gluon correlation functions in the transversely polarized

[1] For the contribution from the quark-gluon correlation functions, consistency between this approach and the transverse-momentum-dependent factorization in the region $\Lambda_{\rm QCD} \ll P_T \ll Q$ with the hard scale Q has also been shown for some processes [6, 7, 8].

[2] The D-meson production at large P_T in SIDIS is dominated by the photon-gluon fusion subprosess. Other possible sources of the asymmetry in the D-meson production are associated with the charm quark content in the polarized nucleon, such as the c-quark transversity distribution and the c-quark-gluon correlation functions, but those intrinsic charm contributions are expected to give tiny corrections in the large P_T region which we are interested in this paper.

nucleon. In section 3, we summarize the twist-3 formalism for the three-gluon correlation functions developed in [19]. We mostly follow the notation used in [19], but introduce a slightly different convention for the azimuthal angles and the hard part, which turned out to be more convenient for our purpose. In section 4, we develop a new master formula which connects the three-gluon contribution to the twist-3 single-spin-dependent cross section to a simpler cross section for the $\gamma^* g \to c\bar{c}$ scattering. We also discuss the extension of the formula for higher order corrections. Section 5 is devoted to a brief summary of the outcome of this paper.

2 Three-gluon correlation functions in the transversely-polarized nucleon

As clarified in [23, 24, 19], there are two-independent twist-3 three-gluon correlation functions in the transversely-polarized nucleon, $O(x_1, x_2)$ and $N(x_1, x_2)$, which are the Lorentz-scalar functions of the two momentum fractions x_1 and x_2, defined as

$$O^{\alpha\beta\gamma}(x_1, x_2) = -gi^3 \int \frac{d\lambda}{2\pi} \int \frac{d\mu}{2\pi} e^{i\lambda x_1} e^{i\mu(x_2 - x_1)} \langle pS| d^{bca} F_b^{\beta n}(0) F_c^{\gamma n}(\mu n) F_a^{\alpha n}(\lambda n) |pS\rangle$$

$$= 2iM_N \left[O(x_1, x_2) g^{\alpha\beta} \epsilon^{\gamma pnS} + O(x_2, x_2 - x_1) g^{\beta\gamma} \epsilon^{\alpha pnS} + O(x_1, x_1 - x_2) g^{\gamma\alpha} \epsilon^{\beta pnS} \right], \quad (1)$$

$$N^{\alpha\beta\gamma}(x_1, x_2) = -gi^3 \int \frac{d\lambda}{2\pi} \int \frac{d\mu}{2\pi} e^{i\lambda x_1} e^{i\mu(x_2 - x_1)} \langle pS| i f^{bca} F_b^{\beta n}(0) F_c^{\gamma n}(\mu n) F_a^{\alpha n}(\lambda n) |pS\rangle$$

$$= 2iM_N \left[N(x_1, x_2) g^{\alpha\beta} \epsilon^{\gamma pnS} - N(x_2, x_2 - x_1) g^{\beta\gamma} \epsilon^{\alpha pnS} - N(x_1, x_1 - x_2) g^{\gamma\alpha} \epsilon^{\beta pnS} \right], \quad (2)$$

up to the irrelevant terms of twist-4 and higher, where $F_a^{\alpha n} \equiv F_a^{\alpha\beta} n_\beta$ with $F_a^{\alpha\beta} = \partial^\alpha A_a^\beta - \partial^\beta A_a^\alpha + g f_{abc} A_b^\alpha A_c^\beta$ being the gluon field strength tensor, d^{bca} and f^{bca} are, respectively, the symmetric and anti-symmetric structure constants of the color SU(3) group, and we have suppressed the gauge-link operators which appropriately connect the field strength tensors so as to ensure the gauge invariance. p is the nucleon momentum, S is the transverse spin vector of the nucleon normalized as $S^2 = -1$, and M_N is the nucleon mass so that $O(x_1, x_2)$ and $N(x_1, x_2)$ are dimensionless. In the twist-3 accuracy, p can be regarded as lightlike ($p^2 = 0$) and n is another lightlike vector satisfying $p \cdot n = 1$. To to be specific, we take $p^\mu = (p^+, 0, \mathbf{0}_\perp)$, $n^\mu = (0, n^-, \mathbf{0}_\perp)$ and $S^\mu = (0, 0, \mathbf{S}_\perp)$. Hermiticity, invariance under the transformations P and T, and the permutation symmetry among the participating three gluon-fields imply that $O(x_1, x_2)$ and $N(x_1, x_2)$ are real functions and satisfy the relations,

$$O(x_1, x_2) = O(x_2, x_1), \qquad O(x_1, x_2) = O(-x_1, -x_2),$$

$$N(x_1, x_2) = N(x_2, x_1), \qquad N(x_1, x_2) = -N(-x_1, -x_2). \quad (3)$$

3 Summary of the twist-3 formalism for $ep^\uparrow \to eDX$

3.1 Kinematics

Here, we summarize the kinematics for the SIDIS process,

$$e(\ell) + p^\uparrow(p,S) \to e(\ell') + D(P_h) + X, \tag{4}$$

where the final D-meson has the mass m_h, i.e., $P_h^2 = m_h^2$. This process is described by the five independent Lorentz invariants:

$$S_{ep} = (p+\ell)^2, \quad x_{bj} = \frac{Q^2}{2p\cdot q}, \quad Q^2 = -q^2 = -(\ell-\ell')^2, \quad z_f = \frac{p\cdot P_h}{p\cdot q}, \quad q_T = \sqrt{-q_t^2}, \tag{5}$$

where q_t is the “transverse” component of q defined as

$$q_t^\mu = q^\mu + \left(\frac{m_h^2 p\cdot q}{(p\cdot P_h)^2} - \frac{P_h\cdot q}{p\cdot P_h}\right) p^\mu - \frac{p\cdot q}{p\cdot P_h} P_h^\mu, \tag{6}$$

satisfying $q_t\cdot p = q_t\cdot P_h = 0$. In the actual calculation we work in the hadron frame where the virtual photon and the initial nucleon are collinear, i.e., both move along the z-axis. In this frame, their momenta q and p are given by

$$q^\mu = (q^0, \vec{q}) = (0,0,0,-Q), \qquad p^\mu = \left(\frac{Q}{2x_{bj}}, 0, 0, \frac{Q}{2x_{bj}}\right) . \tag{7}$$

The azimuthal angle of the hadron plane as measured from the xz plane is taken to be χ and thus the momentum of the D-meson is parameterized as

$$P_h^\mu = \frac{z_f Q}{2}\left(1 + \frac{q_T^2}{Q^2} + \frac{m_h^2}{z_f^2 Q^2}, \frac{2q_T}{Q}\cos\chi, \frac{2q_T}{Q}\sin\chi, -1 + \frac{q_T^2}{Q^2} + \frac{m_h^2}{z_f^2 Q^2}\right) . \tag{8}$$

The transverse momentum of the D-meson in this frame is given by $P_{hT} = z_f q_T$, which is true in any frame where the 3-momenta $\vec{q}$ and $\vec{p}$ are collinear. The azimuthal angle of the lepton plane measured from the xz plane is taken to be ϕ and thus the lepton momentum can be parameterized as

$$\ell^\mu = \frac{Q}{2}\left(\cosh\psi, \sinh\psi\cos\phi, \sinh\psi\sin\phi, -1\right) ,$$

$$\ell'^\mu = \frac{Q}{2}\left(\cosh\psi, \sinh\psi\cos\phi, \sinh\psi\sin\phi, 1\right) , \tag{9}$$

where

$$\cosh\psi = \frac{2x_{bj}S_{ep}}{Q^2} - 1 . \tag{10}$$

We parameterize the transverse spin vector of the initial nucleon S^μ as

$$S^\mu = (0, \cos\Phi_S, \sin\Phi_S, 0), \tag{11}$$

with the azimuthal angle Φ_S of $\vec{S}$. Although three azimuthal angles ϕ, χ and Φ_S are defined above, it is obvious that the cross section for $ep^\uparrow \to eDX$ depends on them through only the relative angles $\phi - \chi$ and $\Phi_S - \chi$. Thus, it can be expressed in terms of S_{ep}, x_{bj}, Q^2, z_f, q_T^2, $\phi - \chi$ and $\Phi_S - \chi$ in the above hadron frame. Note that ϕ, χ and Φ_S are invariant under boosts in the $\vec{q}$-direction, so that the cross section presented below is the same in any frame where $\vec{q}$ and $\vec{p}$ are collinear.

With the kinematical variables defined above, the differential cross section for the D-meson production in SIDIS using the unpolarized lepton can be calculated with

$$\frac{d^6\sigma}{dx_{bj}dQ^2dz_f dq_T^2 d\phi d\chi} = \frac{\alpha_{em}^2}{128\pi^4 x_{bj}^2 S_{ep}^2 Q^2} z_f L^{\mu\nu}(\ell, \ell') W_{\mu\nu}(p, q, P_h), \tag{12}$$

where $L^{\mu\nu}(\ell, \ell') = 2(\ell^\mu \ell'^\nu + \ell^\nu \ell'^\mu) - Q^2 g^{\mu\nu}$ is the corresponding leptonic tensor, $W_{\mu\nu}(p, q, P_h)$ is the hadronic tensor in the same normalization as in [19], and $\alpha_{em} = e^2/(4\pi)$ is the QED coupling constant. The single-spin ($\vec{S}$) dependent part in the cross section (12) describes the SSA in the process (4). For this part, one can transform the azimuthal element in the LHS of (12) as $d\phi d\chi \to d\phi d\Phi_S$ and set $\chi = 0$. In fact, in our previous paper [19], ϕ and Φ_S were used as the azimuthal angles, respectively, for the lepton plane and the spin vector measured from the hadron plane by setting $\chi = 0$ from the beginning. In the expression for the differential cross section derived in [19], the differential element "$d\Phi_S$" appearing in the above sense was missing, and thus should be supplied together with the factor $1/(2\pi)$ for the cross section with all the results unchanged.

3.2 Twist-3 cross section

As shown in [19], the contribution from the three-gluon correlation functions to the hadronic tensor $W_{\mu\nu}$ is relevant for having the sizeable SSA in the collinear factorization to describe the large-P_{hT} D-meson production and is represented by the diagrams of the type shown in Fig. 1. Here, the twist-2 fragmentation function $D(z)$ for a c-quark to become the D-meson is already factorized as the upper blob. In the other part of the diagrams, the partonic hard part $S^{abc}_{\mu\nu;\rho\tau\lambda}(k_1, k_2, q, p_c)$, represented by the middle blob, is combined with the correlation functions $\sim \langle A_b^\tau A_c^\lambda A_a^\rho \rangle$ of the gluon fields, $A_a^\rho(\xi)$, in the nucleon (lower blob), where ρ, τ, λ and a, b, c are, respectively, Lorentz and color indices for the relevant three gluon-fields. μ and ν in $S^{abc}_{\mu\nu;\rho\tau\lambda}(k_1, k_2, q, p_c)$ represent the Lorentz indices for the virtual photon. Compared to the conventional photon-gluon fusion subprocess, $\gamma^* g \to c\bar{c}$, an additional gluon participates in the partonic hard part $S^{abc}_{\mu\nu;\rho\tau\lambda}(k_1, k_2, q, p_c)$ and allows us to obtain interfering phase arising from the unpinched pole contribution of an internal propagator contained in $S^{abc}_{\mu\nu;\rho\tau\lambda}(k_1, k_2, q, p_c)$. Owing to the symmetry property of the spin-dependent part of the nucleon matrix elements $\langle A_b^\tau A_c^\lambda A_a^\rho \rangle$ under the P- and T-transformations [19], only such interfering contribution in the hard part $S^{abc}_{\mu\nu;\rho\tau\lambda}(k_1, k_2, q, p_c)$ can give rise to the single-spin-dependent cross section. In the leading order with respect to the QCD coupling constant,

the corresponding contribution to $S^{abc}_{\mu\nu;\rho\tau\lambda}(k_1, k_2, q, p_c)$ can be obtained from the diagrams shown in Fig. 2 (together with their mirror diagrams), where the bar on a propagator indicates that the pole part is to be taken from the propagator; in principle, other propagators can produce similar pole contributions, but we need not consider separately those contributions, which indeed correspond to the diagrams obtained by the permutation of the gluons in the diagrams of Fig. 2. It has been shown in [19] that the total contribution to the single-spin-dependent cross section from Figs. 1, 2 can be expressed in a gauge-invariant form as

$$W_{\mu\nu}(p, q, P_h) = \int \frac{dz}{z^2} D(z) \int \frac{dx_1}{x_1} \int \frac{dx_2}{x_2} \times \left[\left. \frac{\partial S^{abc}_{\mu\nu;\rho\tau\lambda}(k_1, k_2, q, p_c) p^\lambda}{\partial k_2^\sigma} \right|_{k_i = x_i p} \right]^{\text{pole}} \omega^\rho{}_\alpha \, \omega^\tau{}_\beta \, \omega^\sigma{}_\gamma \, \mathcal{M}^{\alpha\beta\gamma}_{F,abc}(x_1, x_2), \quad (13)$$

up to the twist-3 accuracy in the collinear factorization with $k_1 = x_1 p$ and $k_2 = x_2 p$, where $\omega^\rho{}_\alpha = g^\rho{}_\alpha - p^\rho n_\alpha$ and $\mathcal{M}^{\alpha\beta\gamma}_{F,abc}(x_1, x_2)$ denotes the three-gluon lightcone correlation functions defined in terms of the gluon field-strength tensors, as

$$\begin{aligned}\mathcal{M}^{\alpha\beta\gamma}_{F,abc}(x_1, x_2) &= -g i^3 \int \frac{d\lambda}{2\pi} \int \frac{d\mu}{2\pi} e^{i\lambda x_1} e^{i\mu(x_2 - x_1)} \langle pS | F^{\beta n}_b(0) F^{\gamma n}_c(\mu n) F^{\alpha n}_a(\lambda n) | pS \rangle \\ &= \frac{3}{40} d^{abc} O^{\alpha\beta\gamma}(x_1, x_2) - \frac{i}{24} f^{abc} N^{\alpha\beta\gamma}(x_1, x_2), \end{aligned} \quad (14)$$

with $O^{\alpha\beta\gamma}(x_1, x_2)$ and $N^{\alpha\beta\gamma}(x_1, x_2)$ given in (1) and (2). We introduced the notation $[\cdots]^{\text{pole}}$, where $\left[S^{abc}_{\mu\nu;\rho\tau\lambda}(k_1, k_2, q, p_c)\right]^{\text{pole}}$ implies that the bared propagator arising in Fig. 2 (and its mirror diagrams) should be replaced by its pole part $\propto \delta\left((p_c + k_1 - k_2)^2 - m_c^2\right)$ (or $\delta\left((p_c + k_2 - k_1)^2 - m_c^2\right)$), with p_c the momentum of the c-quark fragmenting into the D-meson, $p_c^2 = m_c^2$; this fixes a momentum fraction at $x_1 = x_2$ in the collinear limit $k_i = x_i p$ in (13). p_c is parameterized by the momentum fraction z associated with the fragmentation function $D(z)$, as

$$\begin{aligned} p_c^\mu &= \frac{1}{z} P_h^\mu + \frac{1}{2p \cdot P_h} \left(m_c^2 z - \frac{m_h^2}{z} \right) p^\mu \\ &= \frac{\hat{z} Q}{2} \left(1 + \frac{q_T^2}{Q^2} + \frac{m_c^2}{\hat{z}^2 Q^2}, \frac{2q_T}{Q} \cos\chi, \frac{2q_T}{Q} \sin\chi, -1 + \frac{q_T^2}{Q^2} + \frac{m_c^2}{\hat{z}^2 Q^2} \right), \end{aligned} \quad (15)$$

where $\hat{z} = z_f / z$, and the second line shows the explicit form in the hadron frame with (7), (8). The summation over the c and $\bar{c}$ quark contributions, as well as the corresponding flavor index on $D(z)$ and $S^{abc}_{\mu\nu;\rho\tau\lambda}(k_1, k_2, q, p_c)$, is implicit in (13). Actually, many terms of twist-3, other than those in (13), are generated by the collinear expansion of the diagram of Fig. 1, and appear to be gauge-noninvariant. We emphasize that all these gauge-noninvariant twist-3 terms either cancel or vanish in the cross section, as demonstrated in [19] by the use of Ward identities.

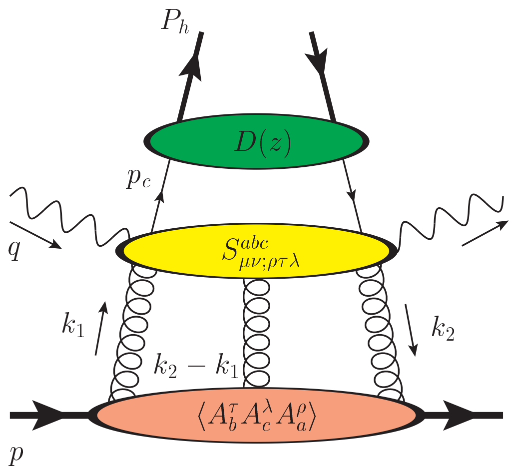

Figure 1: Generic diagram giving rise to the twist-3 contribution to the hadronic tensor of $ep^\uparrow \to eDX$ induced by the gluonic effect in the nucleon. It is decomposed into the nucleon matrix element (lower blob), D-meson matrix element (upper blob), and the partonic hard scattering part by the virtual photon (middle blob).

3.3 Calculation of $L_{\mu\nu}W^{\mu\nu}$

To calculate the contraction $L^{\mu\nu}(\ell,\ell')W_{\mu\nu}(p,q,P_h)$ in (12), we introduce the following four vectors which are orthogonal to each other, similarly as in [19]:

$$\begin{aligned}
T^\mu &= \frac{1}{Q}\left(q^\mu + 2x_{bj}p^\mu\right), \\
X^\mu &= \frac{1}{q_T}\left\{\frac{P_h^\mu}{z_f} - q^\mu - \left(1 + \frac{q_T^2 + m_h^2/z_f^2}{Q^2}\right)x_{bj}p^\mu\right\}, \\
Y^\mu &= \epsilon^{\mu\nu\rho\sigma}Z_\nu X_\rho T_\sigma, \\
Z^\mu &= -\frac{q^\mu}{Q}.
\end{aligned} \tag{16}$$

In the hadron frame specified by (7) and (8), these vectors become $T^\mu = (1,0,0,0)$, $X^\mu = (0,\cos\chi,\sin\chi,0)$, $Y^\mu = (0,-\sin\chi,\cos\chi,0)$, $Z^\mu = (0,0,0,1)$, so that

$$\begin{aligned}
(0,1,0,0) &= \cos\chi X^\mu - \sin\chi Y^\mu, \\
(0,0,1,0) &= \sin\chi X^\mu + \cos\chi Y^\mu,
\end{aligned} \tag{17}$$

In the present case, $W^{\mu\nu}$ can be expanded in terms of the following six independent tensors [19]:

$$\mathcal{V}_1^{\mu\nu} = X^\mu X^\nu + Y^\mu Y^\nu, \qquad \mathcal{V}_2^{\mu\nu} = g^{\mu\nu} + Z^\mu Z^\nu,$$

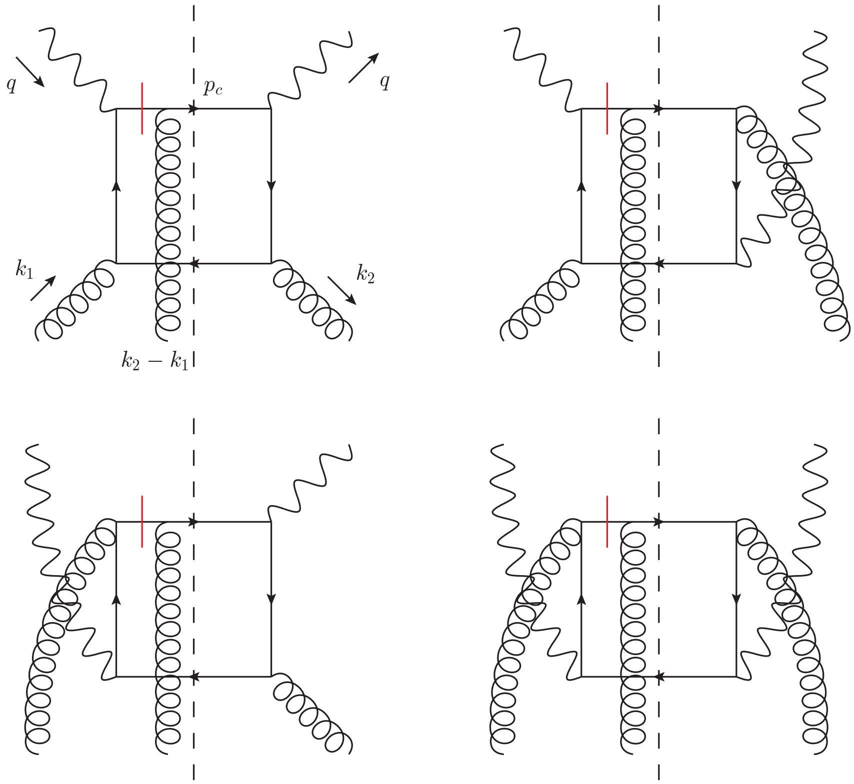

Figure 2: Feynman diagrams for the partonic hard part in Fig. 1, representing the photon-gluon fusion subprocesses that give rise to the pole contribution for $ep^{\uparrow} \to eDX$ in the leading order with respect to the QCD coupling constant. The short bar on the internal c-quark line indicates that the pole part is to be taken from that propagator. In the text, momenta are assigned as shown in the upper-left diagram, where p_c denotes the momentum of the c-quark fragmenting into the D-meson in the final state. The mirror diagrams also contribute.

$$\mathcal{V}_3^{\mu\nu} = T^\mu X^\nu + X^\mu T^\nu, \qquad \mathcal{V}_4^{\mu\nu} = X^\mu X^\nu - Y^\mu Y^\nu,$$
$$\mathcal{V}_8^{\mu\nu} = T^\mu Y^\nu + Y^\mu T^\nu, \qquad \mathcal{V}_9^{\mu\nu} = X^\mu Y^\nu + Y^\mu X^\nu. \tag{18}$$

We also introduce the inverse tensors $\widetilde{\mathcal{V}}_k^{\mu\nu}$ for the above $\mathcal{V}_k^{\mu\nu}$:

$$\widetilde{\mathcal{V}}_1^{\mu\nu} = \frac{1}{2}(2T^\mu T^\nu + X^\mu X^\nu + Y^\mu Y^\nu), \qquad \widetilde{\mathcal{V}}_2^{\mu\nu} = T^\mu T^\nu,$$
$$\widetilde{\mathcal{V}}_3^{\mu\nu} = -\frac{1}{2}(T^\mu X^\nu + X^\mu T^\nu), \qquad \widetilde{\mathcal{V}}_4^{\mu\nu} = \frac{1}{2}(X^\mu X^\nu - Y^\mu Y^\nu),$$
$$\widetilde{\mathcal{V}}_8^{\mu\nu} = \frac{-1}{2}(T^\mu Y^\nu + Y^\mu T^\nu), \qquad \widetilde{\mathcal{V}}_9^{\mu\nu} = \frac{1}{2}(X^\mu Y^\nu + Y^\mu X^\nu). \tag{19}$$

With these definitions, one has

$$L_{\mu\nu}W^{\mu\nu} = \sum_{k=1,\cdots,4,8,9} [L_{\mu\nu}\mathcal{V}_k^{\mu\nu}] \left[W_{\rho\sigma}\widetilde{\mathcal{V}}_k^{\rho\sigma}\right] \equiv Q^2 \sum_{k=1,\cdots,4,8,9} \mathcal{A}_k(\phi-\chi) \left[W_{\rho\tau}\widetilde{\mathcal{V}}_k^{\rho\tau}\right], \tag{20}$$

where $\mathcal{A}_k(\phi-\chi) \equiv L_{\mu\nu}\mathcal{V}_k^{\mu\nu}/Q^2$ parameterize the dependence of the cross section on the azimuthal angle ϕ of the lepton plane relative to the hadron plane (see (9), (8)), and depend on ϕ and χ through $\phi-\chi$, with

$$\begin{aligned}
\mathcal{A}_1(\phi) &= 1+\cosh^2\psi, \\
\mathcal{A}_2(\phi) &= -2, \\
\mathcal{A}_3(\phi) &= -\cos\phi\sinh 2\psi, \\
\mathcal{A}_4(\phi) &= \cos 2\phi\sinh^2\psi, \\
\mathcal{A}_8(\phi) &= -\sin\phi\sinh 2\psi, \\
\mathcal{A}_9(\phi) &= \sin 2\phi\sinh^2\psi.
\end{aligned} \tag{21}$$

By the expansion (20), the cross section for $ep^\uparrow \to eDX$ consists of the five structure functions associated with $\mathcal{A}_{1,2}$, $\mathcal{A}_3$, $\mathcal{A}_4$, $\mathcal{A}_8$ and $\mathcal{A}_9$, respectively, which have different dependences on the azimuthal angle ϕ.

4 Master formula for three-gluon contribution

4.1 Connection between the twist-3 hard part and the $\gamma^* g \to c\bar{c}$ scattering

To obtain the twist-3 cross section based on (13), one has to calculate the corresponding hard part as the derivative, $\partial S^{abc}_{\mu\nu;\alpha\beta\lambda}(k_1,k_2,q,p_c)p^\lambda/\partial k_2^\gamma$, for the contributions from the diagrams in Fig. 2, followed by the collinear limit $k_i \to x_i p$. We note that many building blocks in the diagrams in Fig. 2 depend on k_2 due to the participation of three external gluons with the momenta k_1, k_2 and k_2-k_1, so that taking the derivative with respect to k_2^γ produces many terms in the intermediate step and is quite complicated. As will be shown below, the above hard part for the twist-3 cross section is connected to the hard part with only the two external gluons, representing the "twist-2 level" partonic scattering, $\gamma^* g \to c\bar{c}$. This implies, in particular, some relevant contribution to the hard part for the twist-3 cross section is completely determined by the hard part for the twist-2 unpolarized process $ep \to eDX$, which is associated with the gluon density-distribution in the nucleon, and, similarly, the spin-dependent contributions in the partonic scattering $\gamma^* g \to c\bar{c}$ completely determine the remaining hard part for the twist-3 cross section.

In order to prove this, we proceed similarly as in the proof in [10, 11] for the master formula associated with the quark-gluon correlation, but we discuss each step of our

proof in detail because it involves extensions for the case not only with the three-gluon correlation, but also with the nonzero quark-mass, compared to the massless quark case treated in [10, 11]. We first note that the hard part shown by the diagrams in Fig. 2 has the structure obtained by attaching an extra gluon-line to the c-quark line in the diagrams in Fig. 3, which show the leading-order contribution for the $\gamma^* g \to c\bar{c}$ scattering with the c-quark fragmenting into the final D-meson. We denote the sum of the contributions of the diagrams in Fig. 3 as $S^{(2)ab}_{\mu\nu;\alpha\beta}(xp, q, p_c)$, where the Lorentz indices α and β, as well as the color indices a and b, are associated with the external gluon lines that have the momentum xp. $S^{(2)aa}_{\mu\nu;\alpha\beta}(xp, q, p_c)/8$ with the color indices averaged over represents the hard part for the twist-2 cross sections. In particular, the twist-2 unpolarized cross section in (12) is given by the contribution to the hadronic tensor,

$$W^U_{\mu\nu}(p, q, P_h) = \int \frac{dz}{z^2} D(z) \int \frac{dx}{x} S^{(2)ab}_{\mu\nu;\alpha\beta}(xp, q, p_c) \mathcal{G}^{\alpha\beta}_{ab}(x), \tag{22}$$

where $\mathcal{G}^{\alpha\beta}_{ab}(x)$ is the light-cone correlation function of the gluon's field strength tensors in the nucleon, defined as

$$\mathcal{G}^{\alpha\beta}_{ab}(x) = \frac{1}{x} \int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle pS | F^{\beta n}_b(0) F^{\alpha n}_a(\lambda n) | pS \rangle = -\frac{1}{2} g^{\alpha\beta}_\perp \times \frac{1}{8} \delta_{ab} G(x) + \cdots, \tag{23}$$

with the unpolarized gluon density $G(x)$, and $g^{\alpha\beta}_\perp = g^{\alpha\beta} - p^\alpha n^\beta - n^\alpha p^\beta$. From Fig. 3, $S^{(2)ab}_{\mu\nu;\alpha\beta}(xp, q, p_c)$ is obtained as

$$S^{(2)ab}_{\mu\nu;\alpha\beta}(xp, q, p_c) = \mathrm{Tr}\left[\bar{\mathcal{F}}^b_\beta(xp, q, p_c) \left(\not{p}_c + m_c \right) \mathcal{F}^a_\alpha(xp, q, p_c) \mathcal{D}(xp + q - p_c) \right], \tag{24}$$

where $\mathrm{Tr}[\cdots]$ indicates the trace over both Dirac and color indices, $(\not{p}_c + m_c)$ with (15) is the projection matrix for the D-meson fragmentation function, and the factor,

$$\mathcal{D}(k) \equiv 2\pi(\not{k} - m_c)\delta(k^2 - m_c^2), \tag{25}$$

is associated with the final-state cut for the unobserved $\bar{c}$-quark with the momentum k flowing from the left to the right of the cut. The factor $\mathcal{F}^a_\alpha(xp, q, p_c)$ is the γgcc-vertex function in the left of the cut, containing the photon-quark and the gluon-quark vertices linked by the quark propagator; here, the Lorentz and color indices, α and a, are associated with the external gluon, while the Lorentz index μ for the virtual photon is suppressed for simplicity. The similar factor in the right of the cut can be written as $\bar{\mathcal{F}}^b_\beta(xp, q, p_c) \equiv \gamma^0 [\mathcal{F}^b_\beta(xp, q, p_c)]^\dagger \gamma^0$.

With the replacement $xp \to k_1$ in (24), where k_1 has nonzero transverse components, $S^{(2)ab}_{\mu\nu;\alpha\beta}(xp, q, p_c)$ may be extended to $S^{(2)ab}_{\mu\nu;\alpha\beta}(k_1, q, p_c)$. $S^{abc}_{\mu\nu;\alpha\beta\lambda}(k_1, k_2, q, p_c) p^\lambda$ arising in (13) is obtained by attaching an additional gluon to $S^{(2)ab}_{\mu\nu;\alpha\beta}(k_1, q, p_c)$, in particular, to the c-quark line fragmenting into the D meson, where the gluon carries the momentum $k_2 - k_1$, the polarization associated with p^λ, and the color index c. When the extra gluon is attached in the left of the cut as in Fig. 2, we obtain

$$S^{abc}_{L,\alpha\beta}(k_1, k_2, q, p_c) = \mathrm{Tr}\left[\bar{\mathcal{F}}^b_\beta(k_2, q, p_c) t^c L(k_1 - k_2 + p_c) \mathcal{F}^a_\alpha(k_1, q, k_1 - k_2 + p_c) \mathcal{D}(k_2 + q - p_c) \right], \tag{26}$$

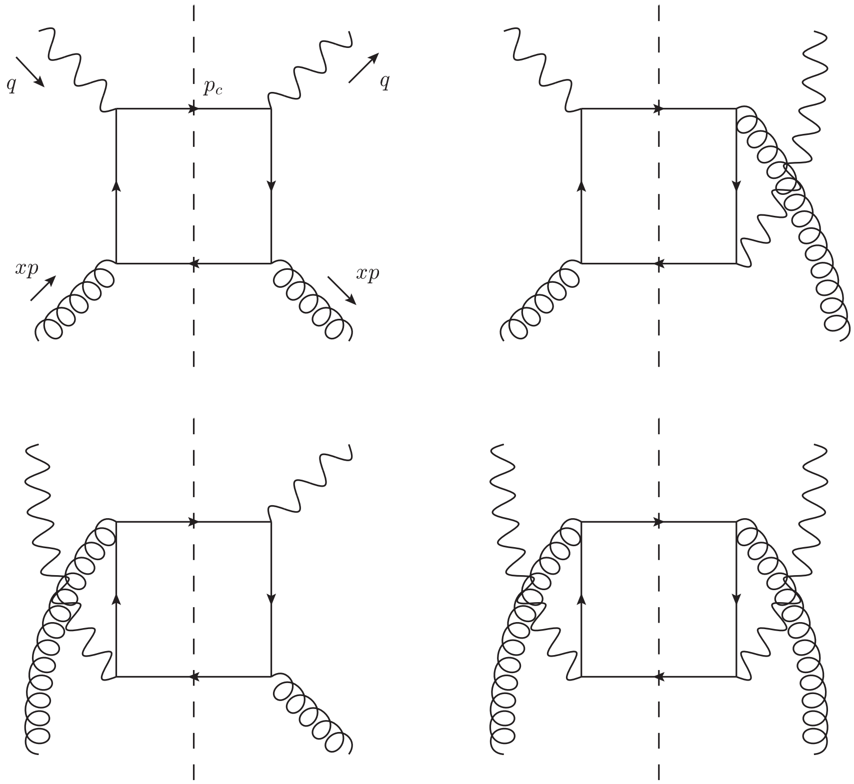

Figure 3: Leading-order diagrams for the $\gamma^* g \to c\bar{c}$ scattering cross section.

where

$$L(k_1 - k_2 + p_c) = (\not{p}_c + m_c)\not{p}\frac{-1}{\not{k}_1 - \not{k}_2 + \not{p}_c - m_c + i\epsilon}, \tag{27}$$

and t^c is the color matrix for the quark-gluon vertex (the coupling constant g for the quark-gluon vertex is absorbed in $\mathcal{M}^{\alpha\beta\gamma}_{F,abc}(x_1, x_2)$ of (14)). Likewise, when the extra gluon is attached in the right of the cut as in the mirror diagrams of Fig. 2, we obtain

$$S^{abc}_{R,\alpha\beta}(k_1, k_2, q, p_c) = \mathrm{Tr}\left[\bar{\mathcal{F}}^b_\beta(k_2, q, k_2 - k_1 + p_c) t^c R(k_2 - k_1 + p_c)\mathcal{F}^a_\alpha(k_1, q, p_c)\mathcal{D}(k_1 + q - p_c)\right], \tag{28}$$

with

$$R(k_2 - k_1 + p_c) = \frac{-1}{\not{k}_2 - \not{k}_1 + \not{p}_c - m_c - i\epsilon}\not{p}(\not{p}_c + m_c) = \gamma^0 L^\dagger(k_2 - k_1 + p_c)\gamma^0, \tag{29}$$

so that

$$S^{abc}_{R,\alpha\beta}(k_1, k_2, q, p_c) = {S^{bac}_{L,\beta\alpha}}^*(k_2, k_1, q, p_c). \tag{30}$$

Thus, $S^{abc}_{\mu\nu;\alpha\beta\lambda}(k_1, k_2, q, p_c)p^\lambda$ can be obtained as the sum of (26) and (28):

$$S^{abc}_{\mu\nu;\alpha\beta\lambda}(k_1, k_2, q, p_c)p^\lambda = S^{abc}_{L,\alpha\beta}(k_1, k_2, q, p_c) + S^{abc}_{R,\alpha\beta}(k_1, k_2, q, p_c). \tag{31}$$

Comparing (24) with (31), we see that $S^{abc}_{\mu\nu;\alpha\beta\lambda}(k_1,k_2,q,p_c)p^\lambda$ is obtained from $S^{(2)ab}_{\mu\nu;\alpha\beta}(k_2,q,p_c)$ by the formal replacements, $(\not{p}_c+m_c)\to t^c L(k_1-k_2+p_c)$ and $(\not{p}_c+m_c)\to t^c R(k_2-k_1+p_c)$, together with the appropriate momentum-shifts in the remaining factors.

We calculate $\partial S^{abc}_{\mu\nu;\alpha\beta\lambda}(k_1,k_2,q,p_c)p^\lambda\ /\partial k_2^\gamma|_{k_i=x_ip}$ in (13) based on (26)-(31), keeping their relevant structure that manifests the above-mentioned correspondence with $S^{(2)ab}_{\mu\nu;\alpha\beta}(k_2,q,p_c)$. For this purpose, we need the collinear limit of (27) and (29),

$$L\left((x_1-x_2)p+p_c\right)=-R\left((x_2-x_1)p+p_c\right)=(\not{p}_c+m_c)\frac{-1}{x_1-x_2+i\epsilon}, \tag{32}$$

and of their derivatives,

$$\left.\frac{\partial L(k_1-k_2+p_c)}{\partial k_2^\alpha}\right|_{k_i=x_ip}=\frac{(\not{p}_c+m_c)\not{p}\gamma_\alpha}{2p\cdot p_c}\frac{1}{x_1-x_2+i\epsilon}-\frac{p_{c\alpha}(\not{p}_c+m_c)}{p\cdot p_c}\frac{1}{(x_1-x_2+i\epsilon)^2}, \tag{33}$$

$$\left.\frac{\partial R(k_2-k_1+p_c)}{\partial k_2^\alpha}\right|_{k_i=x_ip}=\frac{\gamma_\alpha\not{p}(\not{p}_c+m_c)}{2p\cdot p_c}\frac{1}{x_1-x_2+i\epsilon}+\frac{p_{c\alpha}(\not{p}_c+m_c)}{p\cdot p_c}\frac{1}{(x_1-x_2+i\epsilon)^2}. \tag{34}$$

In these relations (32)-(34), the poles relevant to $[\cdots]^{\rm pole}$ in (13) are unveiled; it is straightforward to see that the γgcc-vertex functions and their derivatives do not produce the pole contributions in the $k_i\to x_ip$ limit with $x_i>0$. Based on the pole structures in (32)-(34), we evaluate the pole contribution in (13) as

$$\left[\left.\frac{\partial S^{abc}_{\mu\nu;\alpha\beta\lambda}(k_1,k_2,q,p_c)p^\lambda}{\partial k_2^\gamma}\right|_{k_i=x_ip}\right]^{\rm pole}=\left[\left.\frac{\partial S^{abc}_{\mu\nu;\alpha\beta\lambda}(k_1,k_2,q,p_c)p^\lambda}{\partial k_2^\gamma}\right|_{k_i=x_ip}\right]^{\rm (i)+(ii)+(iii)}, \tag{35}$$

decomposing it into three parts (i)-(iii), where (i) denotes the pole contributions from the second term in (33) and (34), (ii) denotes those from the first term in (33) and (34), and (iii) denotes the remaining pole contributions due to (32). We obtain

$$\begin{aligned}
\left[\left.\frac{\partial S^{abc}_{\mu\nu;\alpha\beta\lambda}(k_1,k_2,q,p_c)p^\lambda}{\partial k_2^\gamma}\right|_{k_i=x_ip}\right]^{\rm (i)}&=\frac{p_{c\gamma}}{p\cdot p_c}\left[\frac{1}{(x_1-x_2+i\epsilon)^2}\right]^{\rm pole}\\
&\times{\rm Tr}\Big[-\bar{\mathcal{F}}^b_\beta(x_2p,q,p_c)t^c(\not{p}_c+m_c)\mathcal{F}^a_\alpha(x_1p,q,(x_1-x_2)p+p_c)\mathcal{D}(x_2p+q-p_c)\\
&+\bar{\mathcal{F}}^b_\beta(x_2p,q,(x_2-x_1)p+p_c)t^c(\not{p}_c+m_c)\mathcal{F}^a_\alpha(x_1p,q,p_c)\mathcal{D}(x_1p+q-p_c)\Big]\\
&=\frac{p_{c\gamma}}{p\cdot p_c}\left[\frac{-1}{x_1-x_2+i\epsilon}\right]^{\rm pole}\\
&\times{\rm Tr}\left[\bar{\mathcal{F}}^b_\beta(x_1p,q,p_c)t^c(\not{p}_c+m_c)\left(p^\mu\frac{\partial}{\partial p_c^\mu}\mathcal{F}^a_\alpha(x_1p,q,p_c)\right)\mathcal{D}(x_1p+q-p_c)\right.
\end{aligned}$$

$$
+\left(p^{\mu}\frac{\partial}{\partial p_c^{\mu}}\bar{\mathcal{F}}^b_{\beta}(x_1p,q,p_c)\right)t^c\left(\not{p}_c+m_c\right)\mathcal{F}^a_{\alpha}(x_1p,q,p_c)\mathcal{D}(x_1p+q-p_c)
$$

$$
+\bar{\mathcal{F}}^b_{\beta}(x_1p,q,p_c)t^c\left(\not{p}_c+m_c\right)\mathcal{F}^a_{\alpha}(x_1p,q,p_c)\left(p^{\mu}\frac{\partial}{\partial p_c^{\mu}}\mathcal{D}(x_1p+q-p_c)\right)\Bigg]
$$

$$
=\frac{p_{c\gamma}}{p\cdot p_c}\left[\frac{-1}{x_1-x_2+i\epsilon}\right]^{\mathrm{pole}}
$$

$$
\times\mathrm{Tr}\left[p^{\mu}\frac{\partial}{\partial p_c^{\mu}}\bar{\mathcal{F}}^b_{\beta}(x_1p,q,p_c)t^c\left(\not{p}_c+m_c\right)\mathcal{F}^a_{\alpha}(x_1p,q,p_c)\mathcal{D}(x_1p+q-p_c)\right.
$$

$$
\left.-\bar{\mathcal{F}}^b_{\beta}(x_1p,q,p_c)t^c\not{p}\,\mathcal{F}^a_{\alpha}(x_1p,q,p_c)\mathcal{D}(x_1p+q-p_c)\right]. \tag{36}
$$

In the second equality in (36), we have performed the Taylor expansion with respect to x_2 around x_1 for the contributions inside the trace Tr, and the partial derivative with respect to p_c implies the shorthand notation of

$$
\frac{\partial}{\partial p_c^{\mu}}f(p_c)\equiv\left.\frac{\partial}{\partial r^{\mu}}f(r)\right|_{r\to p_c}, \tag{37}
$$

for a function $f(r)$ of a four-vector r_{μ}. The leading-order (zeroth-order) term in the Taylor expansion would give rise to the double pole contribution to (36), but the corresponding contributions generated from the two terms in the RHS of (31) cancel; this type of cancellation may be considered as a result of gauge invariance [10]. On the other hand, the second and higher-order terms in the expansion give the vanishing contribution to (36) when combined with the factor $[1/(x_1-x_2+i\epsilon)^2]^{\mathrm{pole}}$. Similarly, for the contribution labeled by (ii) above, we easily obtain

$$
\left[\left.\frac{\partial S^{abc}_{\mu\nu;\alpha\beta\lambda}(k_1,k_2,q,p_c)p^{\lambda}}{\partial k_2^{\gamma}}\right|_{k_i=x_ip}\right]^{\mathrm{(ii)}}
$$

$$
=\left[\frac{1}{x_1-x_2+i\epsilon}\right]^{\mathrm{pole}}\mathrm{Tr}\left[\bar{\mathcal{F}}^b_{\beta}(x_1p,q,p_c)t^c\left(\gamma_{\gamma}-\frac{p_{c\gamma}}{p\cdot p_c}\not{p}\right)\mathcal{F}^a_{\alpha}(x_1p,q,p_c)\mathcal{D}(x_1p+q-p_c)\right]. \tag{38}
$$

The contribution labeled by (iii) can be expressed as

$$
\left[\left.\frac{\partial S^{abc}_{\mu\nu;\alpha\beta\lambda}(k_1,k_2,q,p_c)p^{\lambda}}{\partial k_2^{\gamma}}\right|_{k_i=x_ip}\right]^{\mathrm{(iii)}}
$$

$$
=\left[\frac{1}{x_1-x_2+i\epsilon}\right]^{\mathrm{pole}}\mathrm{Tr}\left[\frac{\partial}{\partial p_c^{\gamma}}\bar{\mathcal{F}}^b_{\beta}(x_1p,q,p_c)t^c\left(\not{p}_c+m_c\right)\mathcal{F}^a_{\alpha}(x_1p,q,p_c)\mathcal{D}(x_1p+q-p_c)\right.
$$

$$
\left.-\bar{\mathcal{F}}^b_{\beta}(x_1p,q,p_c)t^c\gamma_{\gamma}\,\mathcal{F}^a_{\alpha}(x_1p,q,p_c)\mathcal{D}(x_1p+q-p_c)\right], \tag{39}
$$

where we have replaced the derivative $\partial/\partial k_2^\gamma$ by the relevant $\partial/\partial p_c^\gamma$, and have set $x_2 = x_1$ inside the Tr due to the presence of the factor $[1/(x_1 - x_2 + i\epsilon)]^{\rm pole}$. The sum of the results (36)-(39) yields the compact form for (35),

$$\left[\left.\frac{\partial S^{abc}_{\mu\nu;\alpha\beta\lambda}(k_1,k_2,q,p_c)p^\lambda}{\partial k_2^\gamma}\right|_{k_i=x_ip}\right]^{\rm pole} = -i\pi\delta(x_1-x_2)\left(\frac{\partial}{\partial p_c^\gamma} - \frac{p_{c\gamma}p^\mu}{p\cdot p_c}\frac{\partial}{\partial p_c^\mu}\right)\widetilde{S}^{abc}_{\mu\nu;\alpha\beta}(x_1p,q,p_c), \tag{40}$$

where

$$\widetilde{S}^{abc}_{\mu\nu;\alpha\beta}(x_1p,q,p_c) = {\rm Tr}\left[\bar{\mathcal{F}}^b_\beta(x_1p,q,p_c)t^c\left(\not{p}_c + m_c\right)\mathcal{F}^a_\alpha(x_1p,q,p_c)\mathcal{D}(x_1p+q-p_c)\right]. \tag{41}$$

Noting that the RHS of (40) vanishes when contracted by p^γ, (40) may be calculated as

$$\left[\left.\frac{\partial S^{abc}_{\mu\nu;\alpha\beta\lambda}(k_1,k_2,q,p_c)p^\lambda}{\partial k_2^\gamma}\right|_{k_i=x_ip}\right]^{\rm pole} = -i\pi\delta(x_1-x_2)\frac{d}{dp_c^\gamma}\widetilde{S}^{abc}_{\mu\nu;\alpha\beta}(x_1p,q,p_c), \tag{42}$$

where the derivative is to be taken under the on-shell condition $p_c^2 = m_c^2$, regarding p_c^+ as a variable dependent on p_c^- and $p_c^{1,2}$ as in (15). Comparing (41) and (24), $\widetilde{S}^{abc}_{\mu\nu;\alpha\beta}(x_1p,q,p_c)$ is the same as $S^{(2)ab}_{\mu\nu;\alpha\beta}(x_1p,q,p_c)$, except that the former has an extra insertion of the color matrix t^c. The relations (40) and (42) clearly indicate that the three-gluon contribution to the twist-3 cross section for $ep^\uparrow \to eDX$ (see (13)) can be derived from the cross sections for the $\gamma^* g \to c\bar{c}$ scattering, and we find that those relations, obtained for the three-gluon contribution and with a massive quark, have the structure formally similar to the corresponding relations derived in [10, 11] for the quark-gluon contribution and with the massless quarks. Substituting (13), (42) into (12), we obtain the master formula, the main result of this paper, which allows us to derive the explicit form of the whole contribution to the twist-3 SSA in $ep^\uparrow \to eDX$, as we demonstrate in the next section.

4.2 Calculation of the twist-3 cross section based on master formula

We are now in a position to carry out the derivative arising in (42). In the formula (13) with (14) and (42), it is sufficient to consider the corresponding derivative for the color-averaged components of $\widetilde{S}^{abc}_{\mu\nu;\alpha\beta}(xp,q,p_c)$, which coincide with the color-averaged component of $S^{(2)ab}_{\mu\nu;\alpha\beta}(xp,q,p_c)$, as apparent from the color structure in (24), (41). We thus define the color-averaged hard part $S^{(2)}_{\mu\nu;\alpha\beta}(xp,q,p_c)$ as the appropriate color contraction:

$$\frac{1}{8}\delta^{ab}S^{(2)ab}_{\mu\nu;\alpha\beta}(xp,q,p_c) \equiv \frac{1}{2}S^{(2)}_{\mu\nu;\alpha\beta}(xp,q,p_c). \tag{43}$$

Then, we have

$$\frac{3}{40}d^{abc}\widetilde{S}^{abc}_{\mu\nu;\alpha\beta}(xp,q,p_c) = \frac{-i}{24}f^{abc}\widetilde{S}^{abc}_{\mu\nu;\alpha\beta}(xp,q,p_c) = \frac{1}{4}S^{(2)}_{\mu\nu;\alpha\beta}(xp,q,p_c). \tag{44}$$

Furthermore, the above derivative can be calculated most conveniently after taking the contraction of $S^{(2)}_{\mu\nu;\alpha\beta}(xp,q,p_c)$ with the leptonic tensor $L^{\mu\nu}$, as implied in (12). According to (20), we introduce

$$\widetilde{\mathcal{V}}_k^{\mu\nu} S^{(2)}_{\mu\nu;\alpha\beta}(xp,q,p_c) \equiv 2\pi \frac{g^2}{\widehat{z}Q^2} H^k_{\alpha\beta}(xp,q,p_c). \tag{45}$$

Here, in the RHS, we separated the coupling constant, contained in the γgcc-vertex function $\mathcal{F}^a_\alpha(xp,q,p_c)$ in (24), as well as the factor $2\pi/(\widehat{z}Q^2)$ contained in $\mathcal{D}(xp+q-p_c)$ for the unobserved final-state, such that (see (25))

$$2\pi\delta\left((xp+q-p_c)^2) - m_c^2\right) = 2\pi \frac{1}{\widehat{z}Q^2}\delta\left(\frac{q_T^2}{Q^2} - \left(1-\frac{1}{\hat{x}}\right)\left(1-\frac{1}{\hat{z}}\right) + \frac{m_c^2}{\hat{z}^2Q^2}\right), \tag{46}$$

with $\widehat{x} = x_{bj}/x$ and $\widehat{z} = z_f/z$. Thus, one obtains

$$L^{\mu\nu}W_{\mu\nu} = \frac{2\pi g^2}{z_f}\frac{1}{4}\sum_{k=1,\cdots,4,8,9}\int\frac{dz}{z}D(z)\int\frac{dx}{x^2}(-i\pi)\frac{d}{dp_c^\gamma}\left[\mathcal{A}_k(\phi-\chi)H^k_{\alpha\beta}(xp,q,p_c)\right]$$
$$\times\left(O^{\alpha\beta\gamma}(x,x) + N^{\alpha\beta\gamma}(x,x)\right), \tag{47}$$

for the single-spin-dependent contribution, where (see (1) and (2))

$$O^{\alpha\beta\gamma}(x,x) = 2iM_N\left[O(x,x)g_\perp^{\alpha\beta}\epsilon^{\gamma pnS} + O(x,0)(g_\perp^{\beta\gamma}\epsilon^{\alpha pnS} + g_\perp^{\gamma\alpha}\epsilon^{\beta pnS})\right],$$
$$N^{\alpha\beta\gamma}(x,x) = 2iM_N\left[N(x,x)g_\perp^{\alpha\beta}\epsilon^{\gamma pnS} - N(x,0)(g_\perp^{\beta\gamma}\epsilon^{\alpha pnS} + g_\perp^{\gamma\alpha}\epsilon^{\beta pnS})\right], \tag{48}$$

up to the irrelevant terms corresponding to the twist higher than three. We note that (47) with (48) is described by the two kinds of partonic hard parts: the hard part associated with $O(x,x)$ is same as that for $N(x,x)$, but $O(x,0)$ as well as $N(x,0)$ accompanies the hard part of another kind.

The result (47) shows that only the derivative with respect to the transverse components, $(p_c^1, p_c^2) = \mathbf{p}_{c\perp}$, contributes to the twist-3 cross section. As noted below (42), those components can be varied as independent variables when performing the derivative, and, based on the representation in (15), the corresponding derivative may be performed through the magnitude $p_{c\perp} \equiv |\mathbf{p}_{c\perp}| = \widehat{z}q_T$ and the azimuthal angle χ of the transverse components, as

$$\frac{\partial}{\partial p_c^1} = \cos\chi\frac{\partial}{\partial p_{c\perp}} - \frac{\sin\chi}{p_{c\perp}}\frac{\partial}{\partial\chi}, \qquad \frac{\partial}{\partial p_c^2} = \sin\chi\frac{\partial}{\partial p_{c\perp}} + \frac{\cos\chi}{p_{c\perp}}\frac{\partial}{\partial\chi}. \tag{49}$$

The formulae (21) indicate that $\partial/\partial p_{c\perp}$ hits only $H^k_{\alpha\beta}(xp,q,p_c)$ in (47). Substituting (48) into (47) and using $\epsilon^{1pnS} = \sin\Phi_S$ and $\epsilon^{2pnS} = -\cos\Phi_S$, the hard cross section for

$\{O(x,x), N(x,x)\}$ can be obtained as

$$\frac{d}{dp_c^\gamma}\left\{\mathcal{A}_k(\phi-\chi)H^k_{\alpha\beta}\right\}g_\perp^{\alpha\beta}\epsilon^{\gamma pnS}$$

$$=\frac{\sin(\Phi_S-\chi)}{\widehat{z}}\mathcal{A}_k(\phi-\chi)\frac{\partial H^k_{\alpha\beta}g_\perp^{\alpha\beta}}{\partial q_T}+\frac{\cos(\Phi_S-\chi)}{\widehat{z}q_T}\frac{\partial\mathcal{A}_k(\phi-\chi)}{\partial\phi}H^k_{\alpha\beta}g_\perp^{\alpha\beta}, \quad (50)$$

where we have used the fact that the scalar function $H^k_{\alpha\beta}g_\perp^{\alpha\beta}$ does not depend on the angular variable χ. Indeed, comparing with (22), (23), we remark that $-H^k_{\alpha\beta}g_\perp^{\alpha\beta}$ is nothing but the twist-2 partonic part for the unpolarized gluon density $G(x)$, and its explicit form is calculated in [19]. Accordingly, the partonic hard part for $\{O(x,x), N(x,x)\}$ is completely determined from the knowledge on the twist-2 unpolarized cross section. Using the relations $\frac{\partial\mathcal{A}_3}{\partial\phi}=-\mathcal{A}_8$ and $\frac{\partial\mathcal{A}_4}{\partial\phi}=-2\mathcal{A}_9$, we find that the terms associated with the azimuthal structures $\mathcal{A}_{8,9}$ arise from the second term of (50), accompanying $\cos(\Phi_S-\chi)$, although such azimuthal structures were absent from the twist-2 unpolarized cross section (22), i.e., $H^k_{\alpha\beta}g_\perp^{\alpha\beta}=0$ for $k=8,9$ (see (56) below). These remarkable features revealed in (50) have been observed similarly in the master formula for the SGP contribution to the SSA induced by the quark-gluon correlation [10]. We also remind that the result (50) depends on the azimuthal angles through $\phi-\chi$ and $\Phi_S-\chi$, as noted in section 3.1. [3]

The hard part for $\{O(x,0), N(x,0)\}$ can be calculated similarly. In this case, however, we encounter the non-scalar components of $H^k_{\alpha\beta}$, which have the dependence on the angular variable χ. The χ dependence of this type was irrelevant for the cases discussed in [10, 11], but gives the novel contribution in the master-formula approach for the present case. In order to deal with those non-scalar components, we write down the corresponding hard part explicitly as

$$\frac{d}{dp_c^\gamma}\left\{\mathcal{A}_k(\phi-\chi)H^k_{\alpha\beta}\right\}\left(g_\perp^{\beta\gamma}\epsilon^{\alpha pnS}+g_\perp^{\alpha\gamma}\epsilon^{\beta pnS}\right)$$

$$=-2\frac{\partial}{\partial p_c^1}\mathcal{A}_k(\phi-\chi)H^k_{11}\epsilon^{1pnS}-\frac{\partial}{\partial p_c^1}\mathcal{A}_k(\phi-\chi)(H^k_{12}+H^k_{21})\epsilon^{2pnS}$$

$$-\frac{\partial}{\partial p_c^2}\mathcal{A}_k(\phi-\chi)(H^k_{12}+H^k_{21})\epsilon^{1pnS}-2\frac{\partial}{\partial p_c^2}\mathcal{A}_k(\phi-\chi)H^k_{22}\epsilon^{2pnS}$$

$$=-2\sin\Phi_S\left(\cos\chi\frac{\partial}{\partial p_{c\perp}}-\frac{\sin\chi}{p_{c\perp}}\frac{\partial}{\partial\chi}\right)\mathcal{A}_k(\phi-\chi)H^k_{11}$$

[3] Since this dependence is obvious from the beginning, one may alternatively consider the derivative in (47) with (49) at $\chi=0$ to get Φ_S-dependence and restore the χ-dependence by the shift $\Phi_S\to\Phi_S-\chi$ to reach (50), which is much simpler. In other words, one can calculate the cross section for $\chi\to 0$ by regarding χ as the small parameter that allows us to take the derivative d/dp_c^γ in (47). This also applies to (52) below.

$$+\cos\Phi_S\left(\cos\chi\frac{\partial}{\partial p_{c\perp}}-\frac{\sin\chi}{p_{c\perp}}\frac{\partial}{\partial\chi}\right)\mathcal{A}_k(\phi-\chi)(H_{12}^k+H_{21}^k)$$

$$-\sin\Phi_S\left(\sin\chi\frac{\partial}{\partial p_{c\perp}}+\frac{\cos\chi}{p_{c\perp}}\frac{\partial}{\partial\chi}\right)\mathcal{A}_k(\phi-\chi)(H_{12}^k+H_{21}^k)$$

$$+2\cos\Phi_S\left(\sin\chi\frac{\partial}{\partial p_{c\perp}}+\frac{\cos\chi}{p_{c\perp}}\frac{\partial}{\partial\chi}\right)\mathcal{A}_k(\phi-\chi)H_{22}^k, \tag{51}$$

and thus we need the χ dependence of H_{11}^k, $H_{12}^k+H_{21}^k$ and H_{22}^k. This dependence can be separated using (17) like $H_{12}^k=H_{\alpha\beta}^k\left(\cos\chi X^\alpha-\sin\chi Y^\alpha\right)\left(\sin\chi X^\beta+\cos\chi Y^\beta\right)$; as apparent from the definition (16) for the basis vectors constructed with the momenta that are associated with the hadron plane, $H_{XX}^k\equiv H_{\mu\nu}^k X^\mu X^\nu$, $H_{XY}^k\equiv H_{\mu\nu}^k X^\mu Y^\nu$, *etc*, are unchanged by the rotation of the hadron plane around the z-axis, i.e., are independent of χ. After some algebra, one obtains, in the obvious notation,

$$\begin{aligned}
&\frac{d}{dp_c^\gamma}\left\{\mathcal{A}_k(\phi-\chi)H_{\alpha\beta}^k\right\}\left(g_\perp^{\beta\gamma}\epsilon^{\alpha pnS}+g_\perp^{\alpha\gamma}\epsilon^{\beta pnS}\right)\\
&=\sin(\Phi_S-\chi)\left[\frac{-2\mathcal{A}_k(\phi-\chi)}{\widehat{z}}\frac{\partial H_{XX}^k}{\partial q_T}\right.\\
&\qquad\left.+\frac{1}{\widehat{z}q_T}\left(\frac{\partial\mathcal{A}_k(\phi-\chi)}{\partial\phi}H_{(XY+YX)}^k+2\mathcal{A}_k(\phi-\chi)H_{(-XX+YY)}^k\right)\right]\\
&+\cos(\Phi_S-\chi)\left[\frac{\mathcal{A}_k(\phi-\chi)}{\widehat{z}}\frac{\partial H_{(XY+YX)}^k}{\partial q_T}\right.\\
&\qquad\left.+\frac{1}{\widehat{z}q_T}\left(-2\frac{\partial\mathcal{A}_k(\phi-\chi)}{\partial\phi}H_{YY}^k+2\mathcal{A}_k(\phi-\chi)H_{(XY+YX)}^k\right)\right].
\end{aligned}\tag{52}$$

To proceed further, we define the partonic hard cross sections for the "$\gamma^* g\to c\bar{c}$" scattering, separating the delta function of (46) and the electric charge $e_c=2/3$ of the c-quark from the hard parts arising in (50), (52):

$$-H_{\alpha\beta}^k g_\perp^{\alpha\beta}=e_c^2\hat{\sigma}_k^U(Q,q_T,\widehat{x},\widehat{z})\delta\left(\frac{q_T^2}{Q^2}-\left(1-\frac{1}{\hat{x}}\right)\left(1-\frac{1}{\hat{z}}\right)+\frac{m_c^2}{\hat{z}^2Q^2}\right),$$

$$-H_{XX}^k=e_c^2\hat{\sigma}_k^{XX}(Q,q_T,\widehat{x},\widehat{z})\delta\left(\frac{q_T^2}{Q^2}-\left(1-\frac{1}{\hat{x}}\right)\left(1-\frac{1}{\hat{z}}\right)+\frac{m_c^2}{\hat{z}^2Q^2}\right),$$

$$-H_{YY}^k=e_c^2\hat{\sigma}_k^{YY}(Q,q_T,\widehat{x},\widehat{z})\delta\left(\frac{q_T^2}{Q^2}-\left(1-\frac{1}{\hat{x}}\right)\left(1-\frac{1}{\hat{z}}\right)+\frac{m_c^2}{\hat{z}^2Q^2}\right),$$

$$-H^k_{(XY+YX)} = e_c^2 \hat{\sigma}_k^{\{XY\}}(Q, q_T, \widehat{x}, \widehat{z})\delta\left(\frac{q_T^2}{Q^2} - \left(1 - \frac{1}{\hat{x}}\right)\left(1 - \frac{1}{\hat{z}}\right) + \frac{m_c^2}{\hat{z}^2 Q^2}\right), \tag{53}$$

where the partonic cross sections $\hat{\sigma}_k^j$ ($j = U, XX, YY, \{XY\}$) are the functions of Q, q_T, $\widehat{x}$ and $\widehat{z}$. When (50) and (52) using these forms are inserted into (47), the derivative $\partial/\partial q_T$ hitting the delta function in (53) can be treated by integration by parts with respect to x; for example, the contribution from the first term in the RHS of (50), convoluted with the correlation function $O(x, x)$, can be calculated as

$$\begin{aligned}
&\frac{1}{\widehat{z}}\frac{\partial}{\partial q_T}\int \frac{dx}{x^2}\hat{\sigma}_k^U(Q, q_T, \widehat{x}, \widehat{z})O(x, x)\delta\left(\frac{q_T^2}{Q^2} - \left(1 - \frac{1}{\hat{x}}\right)\left(1 - \frac{1}{\hat{z}}\right) + \frac{m_c^2}{\hat{z}^2 Q^2}\right)\\
&= \frac{2q_T}{Q^2}\int \frac{dx}{x^2}\left[\frac{\widehat{x}\hat{\sigma}_k^U(Q, q_T, \widehat{x}, \widehat{z})}{1 - \widehat{z}}\left(x\frac{dO(x, x)}{dx} - 2O(x, x)\right) + \left\{\frac{Q^2}{\widehat{z}}\frac{\partial \hat{\sigma}_k^U(Q, q_T, \widehat{x}, \widehat{z})}{\partial q_T^2}\right.\right.\\
&\quad \left.\left. - \frac{\widehat{x}^2}{1 - \widehat{z}}\frac{\partial \hat{\sigma}_k^U(Q, q_T, \widehat{x}, \widehat{z})}{\partial \widehat{x}}\right\}O(x, x)\right]\delta\left(\frac{q_T^2}{Q^2} - \left(1 - \frac{1}{\hat{x}}\right)\left(1 - \frac{1}{\hat{z}}\right) + \frac{m_c^2}{\hat{z}^2 Q^2}\right),
\end{aligned} \tag{54}$$

where the derivative, $dO(x, x)/dx$, arises as a result of the integration by parts. Similar results are obtained for the contribution associated with the correlation function $N(x, x)$, as well as for the corresponding contributions from (52) to be combined with $O(x, 0)$ and $N(x, 0)$. This indicates that all four nonperturbative functions of x, $O(x, x)$, $N(x, x)$, $O(x, 0)$ and $N(x, 0)$, contribute both in the derivative and nonderivative forms to the twist-3 SSA, and the partonic hard cross sections convoluted with them are entirely determined from the hard scattering parts for the $\gamma^* g \to c\bar{c}$ scattering which corresponds to the $2 \to 2$ process at the twist-2 level.

Substituting (47) into (12) and using (50), (52)-(54), we obtain the leading-order QCD formula for the single-spin-dependent cross section $\Delta\sigma$ in the SIDIS, $ep^\uparrow \to eDX$, generated from the twist-3 three-gluon correlation functions $O(x_1, x_2)$ and $N(x_1, x_2)$ of (1) and (2) as

$$\begin{aligned}
&\frac{d^6\Delta\sigma}{dx_{bj}dQ^2dz_f dq_T^2 d\phi d\chi}\\
&= \frac{\alpha_{em}^2\alpha_s e_c^2 M_N}{16\pi^2 x_{bj}^2 S_{ep}^2 Q^2}\left(\frac{-\pi}{2}\right)\sum_{k=1,\cdots,4,8,9}\int \frac{dx}{x^2}\int \frac{dz}{z}\delta\left(\frac{q_T^2}{Q^2} - \left(1 - \frac{1}{\hat{x}}\right)\left(1 - \frac{1}{\hat{z}}\right) + \frac{m_c^2}{\hat{z}^2 Q^2}\right)D(z)\\
&\times\left(\sin(\Phi_S - \chi)\mathcal{A}_k\frac{2q_T}{Q^2}\left[\frac{\widehat{x}\hat{\sigma}_k^U}{1 - \widehat{z}}\left(x\frac{dO(x, x)}{dx} - 2O(x, x)\right) + \left(\frac{Q^2}{\widehat{z}}\frac{\partial \hat{\sigma}_k^U}{\partial q_T^2} - \frac{\widehat{x}^2}{1 - \widehat{z}}\frac{\partial \hat{\sigma}_k^U}{\partial \widehat{x}}\right)O(x, x)\right]\right.\\
&\quad + \cos(\Phi_S - \chi)\frac{\partial \mathcal{A}_k}{\partial \phi}\frac{\hat{\sigma}_k^U}{\widehat{z}q_T}O(x, x) + (O(x, x) \to N(x, x))\\
&\quad + \sin(\Phi_S - \chi)\left[-2\mathcal{A}_k\frac{2q_T}{Q^2}\left\{\frac{\widehat{x}\hat{\sigma}_k^{XX}}{1 - \widehat{z}}\left(x\frac{dO(x, 0)}{dx} - 2O(x, 0)\right)\right.\right.
\end{aligned}$$

$$
+\left(\frac{Q^2}{\widehat{z}}\frac{\partial\hat{\sigma}_k^{XX}}{\partial q_T^2}-\frac{\widehat{x}^2}{1-\widehat{z}}\frac{\partial\hat{\sigma}_k^{XX}}{\partial\widehat{x}}\right)O(x,0)\Bigg\}
$$

$$
+\frac{1}{\widehat{z}q_T}\left\{\frac{\partial\mathcal{A}_k}{\partial\phi}\hat{\sigma}_k^{\{XY\}}+2\mathcal{A}_k\left(-\hat{\sigma}_k^{XX}+\hat{\sigma}_k^{YY}\right)\right\}O(x,0)\Bigg]
$$

$$
+\cos(\Phi_S-\chi)\Bigg[\mathcal{A}_k\frac{2q_T}{Q^2}\Bigg\{\frac{\widehat{x}\hat{\sigma}_k^{\{XY\}}}{1-\widehat{z}}\left(x\frac{dO(x,0)}{dx}-2O(x,0)\right)
$$

$$
+\left(\frac{Q^2}{\widehat{z}}\frac{\partial\hat{\sigma}_k^{\{XY\}}}{\partial q_T^2}-\frac{\widehat{x}^2}{1-\widehat{z}}\frac{\partial\hat{\sigma}_k^{\{XY\}}}{\partial\widehat{x}}\right)O(x,0)\Bigg\}
$$

$$
+\frac{1}{\widehat{z}q_T}\left(-2\frac{\partial\mathcal{A}_k}{\partial\phi}\hat{\sigma}_k^{YY}+2\mathcal{A}_k\hat{\sigma}_k^{\{XY\}}\right)O(x,0)\Bigg]+\left(O(x,0)\to -N(x,0)\right)\Bigg), \qquad (55)
$$

where $\alpha_s = g^2/(4\pi)$ is the strong coupling constant, $\mathcal{A}_k \equiv \mathcal{A}_k(\phi-\chi)$, and $\hat{\sigma}_k^j$ ($j = U, XX, YY, \{XY\}$) are defined in (53). It is worth noting the relations $\hat{\sigma}_{8,9}^j = 0$ for $j = XX$, YY and $\hat{\sigma}_{1,2,3,4}^j = 0$ for $j = \{XY\}$, since even number of Y^μ's has to be involved in the contraction to give nonzero $\hat{\sigma}_k^j$. We also remind the relations $\frac{\partial\mathcal{A}_3}{\partial\phi} = -\mathcal{A}_8$, $\frac{\partial\mathcal{A}_4}{\partial\phi} = -2\mathcal{A}_9$ $\frac{\partial\mathcal{A}_8}{\partial\phi} = \mathcal{A}_3$ and $\frac{\partial\mathcal{A}_9}{\partial\phi} = 2\mathcal{A}_4$ in (55). It is straightforward to derive the explicit formulae for $\hat{\sigma}_k^j$ by calculating the diagrams for the $2\to 2$ processes in Fig. 3: For example, $\hat{\sigma}_k^U$ ($= -\hat{\sigma}_k^{XX} - \hat{\sigma}_k^{YY}$) are given in Eq. (81) of [19], where $\hat{\sigma}_k^U$ are nonzero for $k = 1,\cdots,4$, while $\hat{\sigma}_{8,9}^U = 0$. Those determine the gluon contribution to the twist-2 unpolarized cross section $\sigma^{\rm unpol}$ for $ep\to eDX$, as

$$
\frac{d^5\sigma^{\rm unpol}}{dx_{bj}dQ^2dz_f dq_T^2 d\phi} = \frac{\alpha_{em}^2\alpha_s e_c^2}{8\pi x_{bj}^2 S_{ep}^2 Q^2}\frac{1}{4}\sum_{k=1}^{4}\mathcal{A}_k(\phi)\int\frac{dx}{x}\int\frac{dz}{z}\sum_{a=c,\bar{c}}D_a(z)G(x)\hat{\sigma}_k^U
$$

$$
\times\delta\left(\frac{q_T^2}{Q^2}-\left(1-\frac{1}{\hat{x}}\right)\left(1-\frac{1}{\hat{z}}\right)+\frac{m_c^2}{\hat{z}^2Q^2}\right), \qquad (56)
$$

which can be obtained immediately from (22), (23) using (43) and (45), integrating over χ for the fixed $\phi' \equiv \phi-\chi$, and perfoming the formal replacement $\phi'\to\phi$ (i.e., ϕ in (56) is understood to be this ϕ'). We do not show the rather lengthy formulae for the other hard cross sections in (53).

Note that (55) shows the result for the c-quark fragmentation channel according to the diagrams in Fig. 3. The contribution for the $\bar{c}$-quark fragmentation channel, due to the diagrams in Fig. 3 with the direction of the quark lines reversed, can be calculated similarly as above, and yields the formula (55) with the c-quark fragmentation function $D(z)$ replaced by the $\bar{c}$-quark fragmentation function and also with the replacements $O(x,x)\to -O(x,x)$ and $O(x,0)\to -O(x,0)$, reflecting the fact that (1) is associated with the C-odd combination of the gluon operators. Combining this result with (55), we obtain the total

result, which can be recast into the following form:

$$\frac{d^6\Delta\sigma}{dx_{bj}dQ^2dz_f dq_T^2 d\phi d\chi}$$

$$= \frac{\alpha_{em}^2\alpha_s e_c^2 M_N}{16\pi^2 x_{bj}^2 S_{ep}^2 Q^2}\left(\frac{-\pi}{2}\right)\sum_{k=1,\cdots,4,8,9}\mathcal{A}_k\mathcal{S}_k\int\frac{dx}{x}\int\frac{dz}{z}\delta\left(\frac{q_T^2}{Q^2}-\left(1-\frac{1}{\hat{x}}\right)\left(1-\frac{1}{\hat{z}}\right)+\frac{m_c^2}{\hat{z}^2Q^2}\right)$$

$$\times\sum_{a=c,\bar{c}}D_a(z)\left[\delta_a\left\{\left(\frac{d}{dx}O(x,x)-\frac{2O(x,x)}{x}\right)\Delta\hat{\sigma}_k^1+\left(\frac{d}{dx}O(x,0)-\frac{2O(x,0)}{x}\right)\Delta\hat{\sigma}_k^2\right.\right.$$

$$\left.+\frac{O(x,x)}{x}\Delta\hat{\sigma}_k^3+\frac{O(x,0)}{x}\Delta\hat{\sigma}_k^4\right\}$$

$$+\left\{\left(\frac{d}{dx}N(x,x)-\frac{2N(x,x)}{x}\right)\Delta\hat{\sigma}_k^1-\left(\frac{d}{dx}N(x,0)-\frac{2N(x,0)}{x}\right)\Delta\hat{\sigma}_k^2\right.$$

$$\left.\left.+\frac{N(x,x)}{x}\Delta\hat{\sigma}_k^3-\frac{N(x,0)}{x}\Delta\hat{\sigma}_k^4\right\}\right], \tag{57}$$

where $\mathcal{A}_k \equiv \mathcal{A}_k(\phi-\chi)$, and $\mathcal{S}_k$ is defined as $\mathcal{S}_k = \sin(\Phi_S-\chi)$ for $k = 1,2,3,4$ and $\mathcal{S}_k = \cos(\Phi_S-\chi)$ for $k = 8,9$. The quark-flavor index a can, in principle, be c and $\bar{c}$, with $\delta_c = 1$ and $\delta_{\bar{c}} = -1$, so that the cross section for the $\bar{D}$-meson production $ep^\uparrow \to e\bar{D}X$ can be obtained by a simple replacement of the fragmentation function to that for the $\bar{D}$ meson, $D_a(z) \to \bar{D}_a(z)$. Comparing (57) with (55), we find

$$\Delta\hat{\sigma}_k^1 = \frac{2q_T\widehat{x}}{Q^2(1-\widehat{z})}\hat{\sigma}_k^U, \tag{58}$$

and, similarly, the partonic hard cross sections $\Delta\hat{\sigma}_k^j$ ($j = 2,3,4,\ k = 1,\cdots,4,8,9$) are completely expressed by $\hat{\sigma}_k^j$ ($j = U, XX, YY, \{XY\}$) in (53). Substitution of the above-mentioned explicit forms of $\hat{\sigma}_k^j$ into these relations yields the formulae for $\Delta\hat{\sigma}_k^j$; these formulae, of course, agree with Eqs. (71)-(74) in [19], which were obtained by the direct calculation of the three-gluon diagrams in Fig. 2 in our previous study.

4.3 Toward extension to higher orders

As we have demonstrated in the last section, our master formula allows us to derive the explicit form of the twist-3 SSA for $ep^\uparrow \to eDX$ in the leading-order QCD, taking into account the whole contribution induced by the three-gluon correlation inside the transversely-polarized nucleon. Thus, extension of our master formula beyond the leading-order QCD is interesting in that it would provide a powerful framework that allows us to calculate the higher-order corrections to the SSA, which are unknown at present.

To derive the master formula, our starting point was (13), which gives the whole twist-3 cross section as the SGP contribution at $x_1 = x_2$. The derivation of (13) presented in [19] relies on particular properties satisfied by the corresponding partonic scattering amplitudes with an on-shell internal line for which the propagator is replaced by its pole contribution. In particular, those properties include Ward identities for the relevant partonic amplitudes at the leading order. We expect that the similar Ward identities hold even after inclusion of the higher-order corrections employing a suitable gauge choice like the background-field gauge, and that (13) is useful beyond the leading order. Once the cross section is expressed as in (13), it is easy to see that the formulae (40), (42) hold for the SGP contributions: We note that the essential ingredient leading to these relations is the simple correspondence between the hard scattering part (31) relevant to the SGP contributions and the hard part (24) for the $\gamma^* g \to c\bar{c}$. As shown in (41) and (42), for the contributions that can be expressed as (26)-(31) using the γgcc vertices $\mathcal{F}^a_\alpha$ and $\bar{\mathcal{F}}^b_\beta$, the derivative with respect to k_2^γ in (13) can be "transformed" into the derivative with respect to p_c^γ in the master formula, without modifying $\mathcal{F}^a_\alpha$ and $\bar{\mathcal{F}}^b_\beta$. Similarly, for the higher-order contributions that can be expressed generically as (26)-(31) using the γgcc vertices $\mathcal{F}^a_\alpha$ and $\bar{\mathcal{F}}^b_\beta$ with the corresponding corrections included, the result (42) holds without additional modification to $\mathcal{F}^a_\alpha$ and $\bar{\mathcal{F}}^b_\beta$.

When including higher-order corrections, the SGP contributions may also occur from the diagrams that contain, in general, a number of final unobserved partons, as shown in Fig. 4. In that case, the vertex $\mathcal{F}^a_\alpha$ in Fig. 4 has a number of external legs together with the loop correction inside. Still, we note that only the diagrams, which have the extra gluon-line attached to the final parton fragmenting into the D-meson, eventually contribute to the twist-3 cross section [2]. Because of this particular structure in the relevant SGP contributions, we can show that the relation (42) and thus our master formula hold for the twist-3 single-spin-dependent cross section for $ep^\uparrow \to eDX$, even after the inclusion of the higher-order corrections. The details will be discussed elsewhere.

5 Summary

In this paper, we have derived the master formula for the contribution of the three-gluon correlation functions to the twist-3 single-spin-dependent cross section for the D-meson production in SIDIS, $ep^\uparrow \to eDX$. This formula connects the twist-3 effects due to the interference arising in the partonic hard cross section to the Born cross sections for the $\gamma^* g \to c\bar{c}$ scattering at the twist-2 level. In particular, the hard cross sections for the three-gluon correlation functions $\{O(x,x), N(x,x)\}$ are completely determined by the hard cross sections associated with the gluon density distribution in the twist-2 unpolarized cross section for $ep \to eDX$. In the similar master formula derived for the contribution of the twist-3 quark-gluon correlation functions, only the SGP component of the cross section was connected to the twist-2 unpolarized cross section. For the present case with the three-gluon correlation functions, all contributions to the corresponding cross section appear as the SGP contribution, and thus the master formula derived here is for the total twist-3 cross section. The formula derived here can be easily extended to the three-gluon contribution

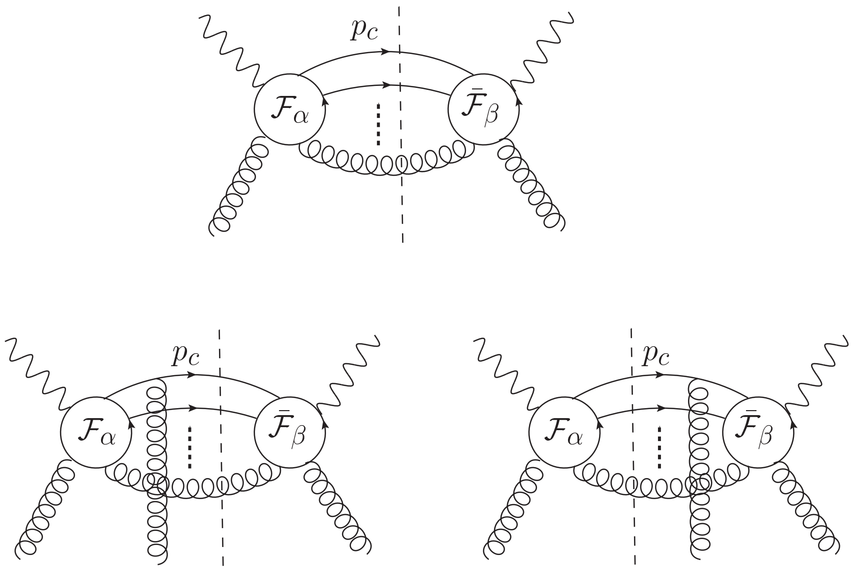

Figure 4: Generic diagrams for the higher-order corrections to the twist-2 unpolarized cross section (upper diagram) and the twist-3 single-spin-dependent cross section (lower diagrams).

to $p^{\uparrow}p \to DX$ [25]. The derivation of the formula is based on the general structure and properties of the relevant twist-3 hard scattering part, which are expected to hold even after including the higher order corrections.

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